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How to Generate Square Wave, Constant Duty Cycle, Transient Response Curves

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APPLICATION NOTE

Abstract

This monograph explains how to construct families of square wave duty cycle transient heating curves based on the "single pulse" transient response. The standard approximate formulations (derived as truncated series solutions from linear superposition) are developed, and resulting error is discussed (including why these formulas are guaranteed to be conservative). Also, the complete infinite series solution is summed assuming a form arising from thermal RC (resistor/capacitor) networks. This results in closed−form expressions for maximum peak steady−state, minimum peak steady−state, and (thus) peak−to−peak steady−state junction temperature excursions. Limitations of the RC−derived solution are discussed, namely fit accuracy and short−time response.

Glossary of Symbols

a	"on" time of duty cycle, or pulse width (see also "t"); also, delay, from start of period until "on'' time, of generalized periodic square pulse
b	delay, from start of period until "off'' time, of generalized periodic square pulse
C's	thermal capacitances in general
d	duty cycle as fraction of unity (=a/p)
F(a,b,p)	response to generalized periodic square pulse
G(t)	generalized periodic power input function
H(a,p,n)	square wave "peak" (relative maximum response) at end of n^{th} cycle
f	frequency of square wave (inverse of its period, i.e., 1/p)
i,j	summation indices
p	period of square wave (inverse of its frequency, i.e. 1/f)
r(t)	normalized single−pulse transient response, having unity value at steady−state
r(t,d)	normalized square wave response, expressed as a function of pulse width and duty cycle
Q,Q_{avg}	power, instantaneous or average
R(t)	single−pulse transient response, having dimensions of thermal resistance
R_∞	steady−state thermal resistance (final value of single−pulse transient response)
R(t,d)	square wave "peak" response, expressed as a function of pulse width and duty cycle
R_i	thermal "resistance" (amplitude) of i^{th} term of RC model single−pulse transient response
R's	thermal resistances in general
t	time, the abscissa of the transient response curve; also, pulse width ("on" time)
τ_i	time constant of i^{th} term of RC model transient solution
V(a,p,n)	square wave "valley" (relative minimum response) at end of n^{th} cycle
Y(t,d)	square wave "valley" response (intercycle minimum at steady−state)
Δ(t,d)	peak−to−peak (i.e., peak to valley) steady−state square wave response

INTRODUCTION AND BACKGROUND FORMULAS

There are two different formulas for peak junction temperature of square wave, constant duty cycle semiconductor operation sometimes found in manufacturer's data sheets – often accompanying a plotted family of "duty cycle" curves on a thermal transient response chart (such as in Figure 1).

$$r(t,d) = d + (1-d) * r(t) \qquad \text{(eq. 1)}$$

$$r(t,d) = d + (1-d) * r\left(t + \frac{t}{d}\right) + r(t) - r\left(\frac{t}{d}\right) \qquad \text{(eq. 2)}$$

Equation 2 is often seen in the form:

$$r(t,d) = d + (1-d) * r(t + p) + r(t) - r(p) \qquad \text{(eq. 3)}$$

where the period, p, is explicit (but may be confusing, as it is not an explicit parameter, but follows from the interrelated definitions of t, d, and p).

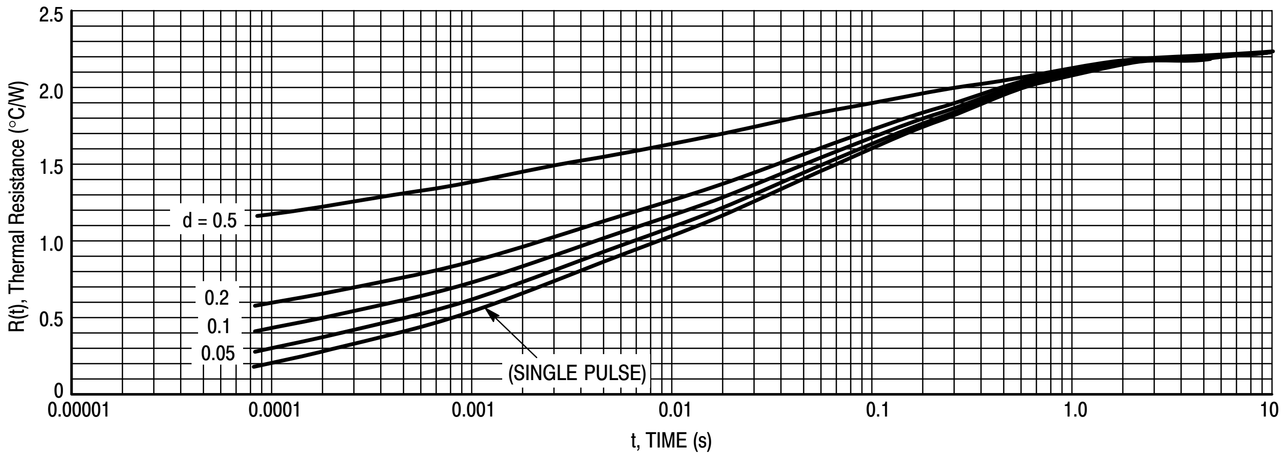


Figure 1. Family of Peak Heating Curves for a Semiconductor Device

As will be shown, these formulas turn out to be “first order” and “second order” approximations to the exact solution to the problem. The problem with these standard formulations is that although they are guaranteed to be conservative (not necessarily in itself a problem), the amount of conservative error is essentially unknown. The problem with the “exact” solution (i.e., the infinite series from which these approximations arise) is that it is computationally very slow. Numerical experiments suggest that the second order formula lowers the first order result by as much as 6%, and that further refinements (i.e., progressively higher−order approximations) probably lower the final result by no more than another couple of percent. However, since the infinite series (exact solution) is non−alternating, the error cannot be easily bounded.

As an alternative, it is often the case that an equivalent thermal RC−network which fits the experimental single−pulse heating curve is available. When this is so, the single−pulse curve has, in effect, been described by a summation of exponential terms having amplitudes and time constants* as follows:

$$R(t) = \sum_{i=1}^{m} R_i\left(1 - e^{-\frac{t}{\tau_i}}\right) \qquad \text{(eq. 4)}$$

Beginning with this exponential expression for the single−pulse heating curve, a relatively simple closed−form solution to the steady−state peak temperature can be derived. It predicts “exact” values quite in line with the computationally intractable infinite series, yet it is fully as fast to calculate as either of the approximate formulas already discussed. Its main drawback, however, is that for times shorter than the fastest time−constant of the model, the RC network is known to significantly underestimate the response; hence the square wave responses are, not surprisingly, equally poor. (Interestingly, perhaps, the limit is correct as the pulse width goes exactly to zero, for any duty cycle. Where it departs significantly is for finite pulse widths between zero and the minimum time constant of the exponential formulation.)

Whatever approach is taken, there are implications for accuracy of results, and efficiency of execution. For instance, Microsoft Excel (or a similar spreadsheet−based computational aid) is readily available to many customers. LabVIEW™ may be available to some, or possibly Mathematica® or MatLab. The following table summarizes the major tradeoffs in approach.

*In passing, we note that these amplitudes represent the resistances of the rungs of a non−grounded−capacitor thermal RC network, and the time constants are the RC products of the R and C values of each rung. However, this type of network, where the C’s are connected between rungs − rather than being attached between each rung and thermal “ground” − has no physical significance, whereas the grounded−capacitor networks have physical meaning. Further, these amplitudes have only a vague correspondence to the resistances of a truly physically significant network. Unfortunately, there is no correspondingly simple mathematical expression for the transient response in terms of the physically significant resistors and capacitors. The justification for the non−grounded−capacitor model is purely the convenience of its simple mathematical expression.

Table 1. Comparison of Square Wave Fixed Duty Cycle Peak Temperature Curve Methods

	First Order	Second Order	Infinite Series	RC−Exact
Accuracy	Conservative/Poorly Bounded	Conservative/Poorly Bounded	Good; Limited by experimental noise	Good; Limited by RC model's accuracy
Computational Speed	Very Fast	Very Fast	Very Slow	Very Fast
Excel Complexity	Easy	Awkward, but it's done already	Intractable without complex macros	Easy
LabVIEW Complexity	Easy	Easy	Not trivial, but it's done already	Easy
Noise Immunity	Good	Sensitive	Sensitive	Good
Main Limitations	Accuracy	Accuracy	Speed and Noise Sensitivity	Short−time response; requires RC model

The Infinite Series Solution

The first thing to do is to convert the "standard" formulas back from a normalized to a dimensionalized form. Though there is a certain elegance to the normalized forms, in practice the only thing you accomplish by putting real package data into a normalized form is that you give the customer another step to perform in using the information – namely, unnormalizing it again so he can make real temperature calculations. Indeed, there is no reason whatsoever to expect that the same curve measured on one device in one particular application, will somehow magically apply to another, different device (or worse, the same device but in a different mounting application), simply by "normalizing" them both to a common value of unity at some arbitrarily chosen time. Perhaps this has been done in the past because the data was so thoroughly derated, safety−factored, and guard−banded, that there was indeed one basic transient curve that applied equally (poorly!) to every semiconductor device. It is certainly *not* to be expected that the same die in two different packages (say a TO92 vs. an SO−8) will follow the same normalized curve just because they both start at zero and end at unity – unless it should happen that the final dimensionalized steady−state value was the same for both packages. What *is* certain, is that the same die in two different packages will follow exactly the same *dimensionalized* transient response for the first few milliseconds, until the heat is out of the silicon and nearby leadframe, and has entered into the differing package structure farther out. Thus, if the final steady−state values *are* different (which is almost as certain), then the normalized responses cannot possibly be the same at the sub−millisecond range.

Equations 1 and 2 can be re−dimensionalized by multiplying through by the steady−state thermal resistance, R_∞.

$$R(t, d) = d \cdot R_\infty + (1-d) \cdot R(t) \qquad \text{(eq. 5)}$$

$$R(t, d) = d \cdot R_\infty + (1-d) \cdot R\left(t + \frac{t}{d}\right) + R(t) - R\left(\frac{t}{d}\right) \qquad \text{(eq. 6)}$$

Given $R(0) = 0$ and $R(\infty) = R_\infty$, it may be observed that both of these equations have as limits:

$$\lim_{t \to 0} R(t, d) = d \cdot R_\infty$$

$$\lim_{t \to \infty} R(t, d) = R_\infty$$

Now let us consider a typical single−pulse transient response (also called the "heating curve"), as shown in Figure 2. This describes the rise in temperature above starting equilibrium ("normalized" in a way, by dividing through by power input) as a function of how long (constant) power has been applied, as in Figure 3. It thus can be seen that this temperature rise represents the effect of a square "pulse" of power, because we're only interested in how hot it gets up through the instant the pulse is turned off.

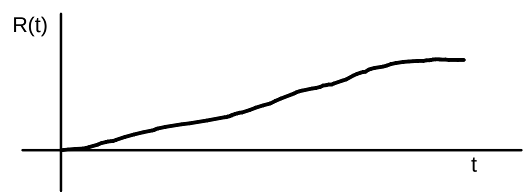


Figure 2. Single Pulse Heating Curve

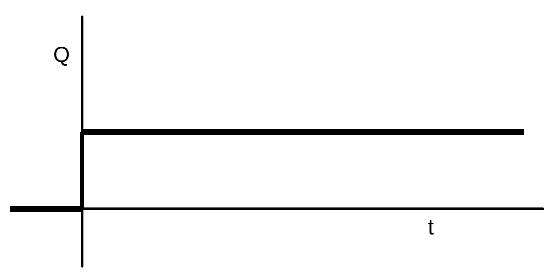


Figure 3. Power Input Corresponding to Single Pulse Heating Curve

Whether the pulse is actually turned off at time t or is left on, the curve has the same value up through that instant. This has the effect of making the single−pulse heating curve the limiting case of a zero−percent duty cycle, as it is generally seen on the charts. In other words, as is easily seen from Equation 4, and less easily from Equation 5,

$$R(t, 0) = R(t) \quad \text{(eq. 7)}$$

Why is this single−pulse heating curve so valuable? There are other fairly “basic” transient responses which could be used to characterize a device (the impulse response, for instance); but the single−pulse response permits us to estimate the response of the device to any other waveform, albeit with a little effort in even moderately complicated situations. It does allow us to address as directly as possible the closely related situation of an infinite train of square waves. All this arises because of the principle of “linear superposition.” For linear superposition to apply, we must be willing to make the following assumptions: (1) certain material properties are constant (that is, density, heat capacity, and thermal conductivity) − at least over the practical temperature ranges with which we concern ourselves in typical semiconductor applications; (2) there is no “internal heat generation” within the various material domains. This latter restriction is significant, though perhaps not so severe as might immediately be thought. What it amounts to in our problem is that heat input to (or exit from) the system can occur only at the boundaries; to wit, the “active” surface of the silicon chip, though buried deeply inside the package, counts as a boundary. Given these assumptions, the Fourier heat conduction equation (a partial differential equation in spatial and time coordinates) is linear. This means that a linear combination of solutions is also a solution. A linear combination means multiplying a solution by a constant (positive or negative), and adding it to (or subtracting it from) any other solution, and obtaining a new result that satisfies the governing equation just as perfectly as did the individual, separate solutions. Since we have the solution to the square−edged constant power input problem, we may thus obtain the solution to any other problem which can be represented by a linear combination of square−edged power inputs. Some examples will clarify the principles.

Example 1: A single pulse which is actually turned off (as opposed to that which generated the single pulse heating curve). Using linear superposition, it should be easy to see that we can generate a single, finite duration pulse, by starting with a constant power applied at the beginning of the pulse, and then, at the appropriate later moment, superimposing a negative−going pulse of equal amplitude − illustrated in Figure 4. So we see that the temperature response will be a similar combination of the single−pulse heating curve starting at time zero, and then subtracting it from itself at a later time.

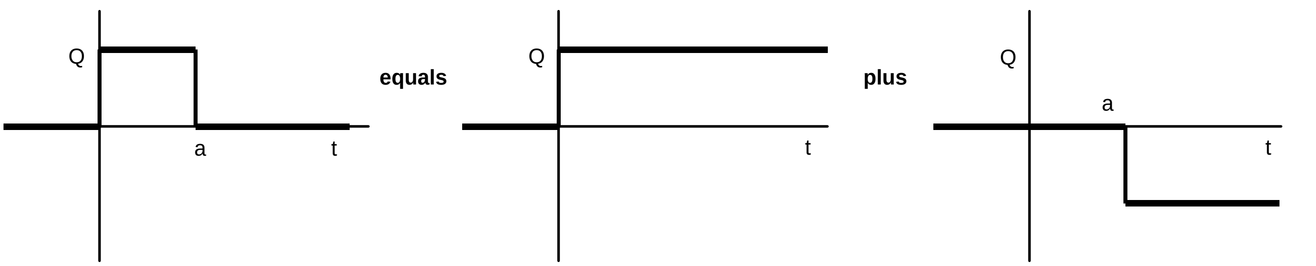


Figure 4. A Finite Pulse of Power, Decomposed into Two Infinite Steps of Constant Power

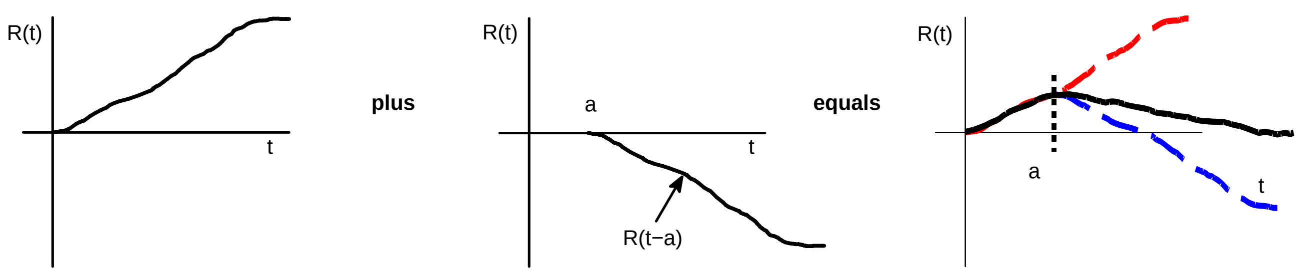


Figure 5. Temperature response of a finite pulse of power (constructed from superposition of two single pulse responses).

Example 2: Two pulses with different amplitudes and durations. Figure 6 illustrates the decomposition of the two pulses into four infinite steps of constant power. Note different amplitudes and starting times.

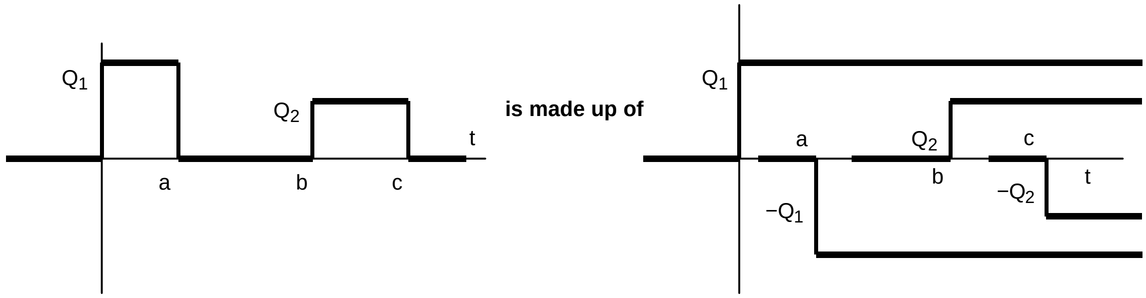


Figure 6. Two Finite Pulses Decomposed into Infinite Steps

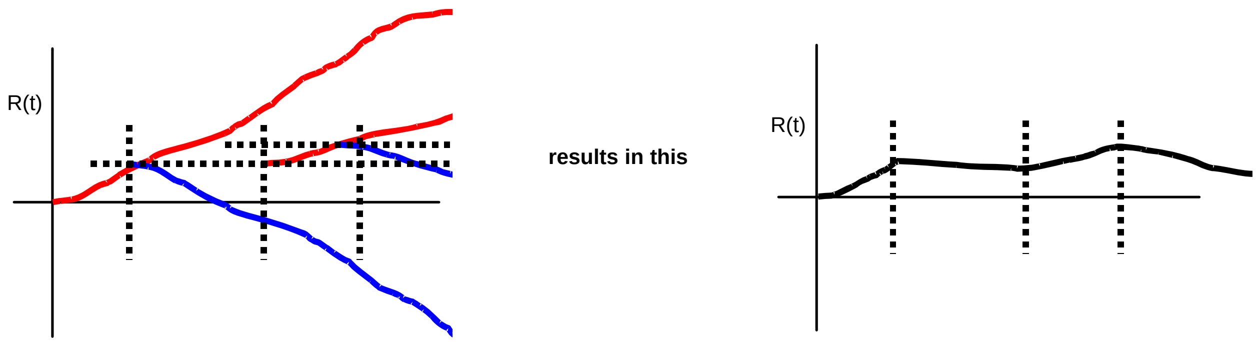


Figure 7. Temperature response constructed for two finite pulses (constructed from superposition of four single pulse responses).

Example 3: A short ramp can be constructed from as many smaller infinite steps as necessary for desired resolution in time or temperature, as illustrated in Figure 8.

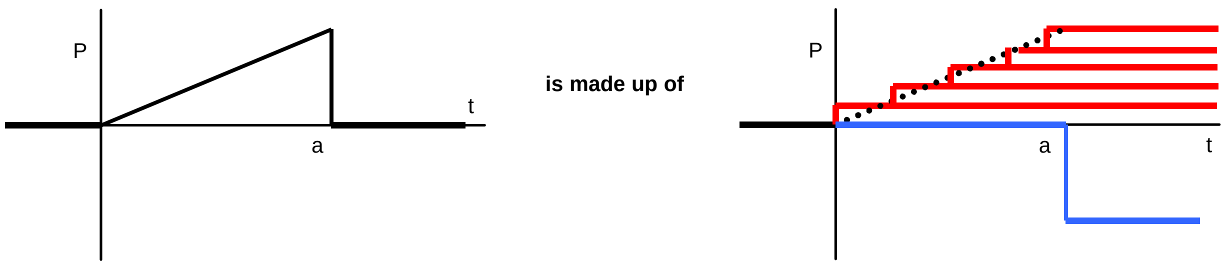


Figure 8. A Finite Ramp Decomposed into Several Infinite Steps

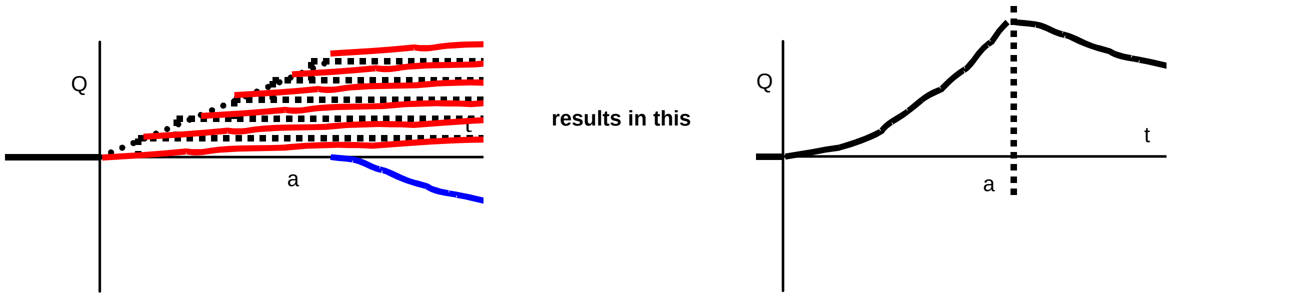


Figure 9. Temperature response constructed for finite ramp (constructed from superposition of many single pulse responses).

For the linear superposition of the small steps in Figure 9, note that the single−pulse response is scaled by a small amount for the small positive contributions, and a full−scale single subtraction is made for the negative going pulse to “turn off” the ramp.

Example 4: An arbitrary pulse; hopefully by this point the superposition technique should contain no surprises. An arbitrary pulse (Figure 10), or train of pulses, is simply handled by starting smaller, incremental steps of power, either positive or negative, at whatever times and with whatever amplitudes are appropriate for approximating the shape to the required resolution and accuracy. Once begun, each step response continues forever. In practice, obviously, this becomes computationally cumbersome, but it is worthwhile to consider before we move on to the final example (i.e., the square wave).

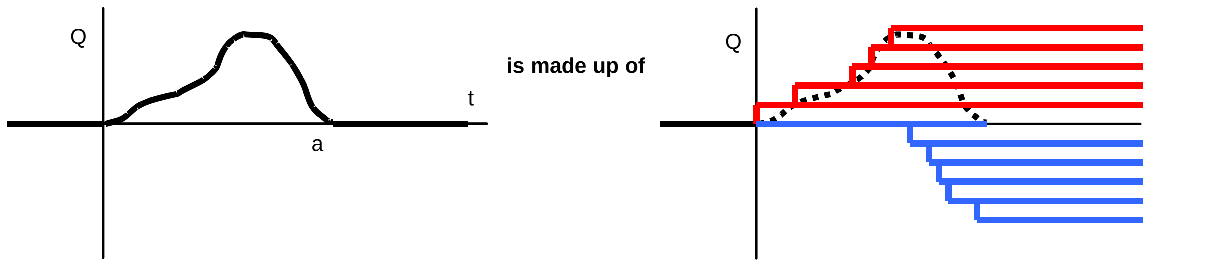


Figure 10. An Arbitrary Pulse Decomposed into Several Infinite Steps

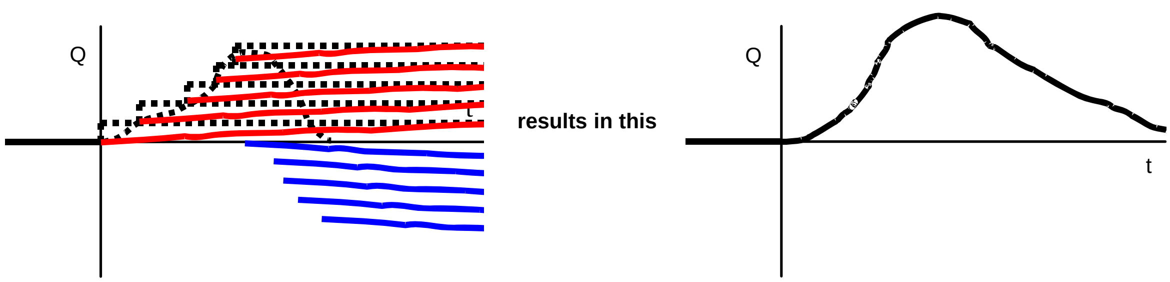


Figure 11. Temperature Response Constructed for Arbitrary Pulse of Figure 10

It may be noted that if the elemental step inputs (hence responses) are made of equal amplitude, but spaced at variable starting intervals so as to best follow the desired input, and if the basic step response is tabulated to reach steady−state at some finite time, then a computational shortcut results. Pairs of positive and negative going responses, once they reach steady−state, cancel out exactly and may be dropped from the list of responses which must be tracked and carried along. If an infinite train of pulses is being modeled, in fact, this list grows infinitely long unless such a shortcut is taken. Alternatively (especially if equal amplitude elements are not convenient), as each elemental response reaches its individual steady−state, its final value can simply be summed into an accumulating steady−state “constant,” and only the elements which have not yet reached steady−state need to be carried along. Consider, however, the massive computations involved in a typical transient response: A tabulated “single−pulse response” curve typically spans several orders of magnitude, for instance, from 1.0 microsecond out to 10 seconds (thus six orders of magnitude). If one is modeling a pulse train whose period is 10 microseconds, then each elemental response will have to be carried along for a million terms until it can be rolled into the accumulating constant value (or cancelled out with an equal and opposite amplitude response). We see therefore, that although the superposition scheme is fundamentally straightforward, it may be computationally unwieldy for any but the most simple (and probably finite) power inputs.

Example 5: The constant duty cycle, infinite square wave. Our final example is actually the problem which we will follow through to specific numerical conclusions, namely an infinite train of square waves comprising a fixed duty cycle power input. We will characterize the train in a couple of interrelated ways.

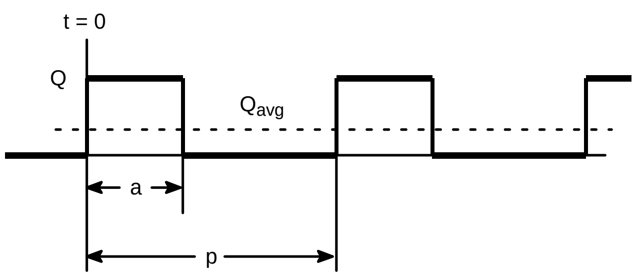


Figure 12. Fixed Duty Cycle Square Wave Defined

Each cycle of the wave has a period p, of which the first portion is the “on” interval a (also called the pulse width). From these quantities we can define the duty cycle factor, d. Equation 8 shows the interrelationships between p, a, and d.

$$d = \frac{a}{p} \quad p = \frac{a}{d} \quad a = d \cdot p \qquad \text{(eq. 8)}$$

Also note that the average power is related to the peak power by the duty cycle, as:

$$Q_{avg} = d \cdot Q \qquad \text{(eq. 9)}$$

For convenience, we shall define time zero to correspond to the rising edge of the first pulse from which we begin our computations. There is also a matter of convention in plotting the resulting responses, i.e., the choice of the horizontal axis. By convention, this time axis, usually denoted t, is the pulse width. So in certain formulas, we may find a t where we might otherwise have found an a, or vice versa. We now may set about calculating the quasi−steady response of an infinite train of such pulses. Two slightly different approaches will be taken.

Method 1 – Pulse train applied to system initially at ambient

The first approach is to begin simply from an unpowered initial state, where the starting temperature is known – namely ambient (and may for convenience be defined as zero, since we’re looking for temperature rises above this value). We then tally up the responses from an ever growing list of individual cycles, until the first pair of plus and minus have reached steady−state and thus cancel each other out exactly. At that point, we must, by numerical definition, be at steady−state, since each subsequent pair of plus and minus responses will also cancel out as we add in two fresh pairs for the next cycle. This process is illustrated in Figure 13. Note that because we’re always dealing with equal amplitudes of positive and negative going steps, we can work directly with the single−pulse transient response function, without worrying about scaling it by any particular power. In effect, we’re computing directly the response to an average power of d, since each pulse represents a peak power of unity.

At the falling edge of the second pulse in the illustration, we can tally up the responses of two previously begun positive responses, and subtract off one previously begun negative response, to get the accumulated “peak” temperature at that instant. Similarly, at the rising edge of the third pulse in the illustration, we can tally up the responses of two previously begun positive responses, and subtract off two previously begun negative responses, to get the accumulated “valley”. So the “peaks” are what come from the summed responses at the falling edges, and the “valleys” will come from the sums at the rising edges. Usually, of course, we’re interested in the peaks.

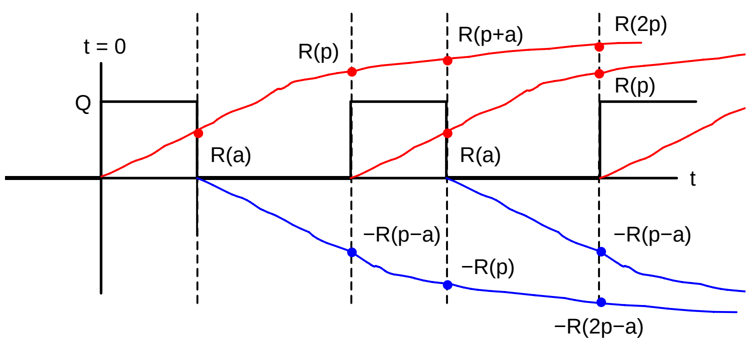


Figure 13. Building Response to Fixed Duty Cycle Square Wave

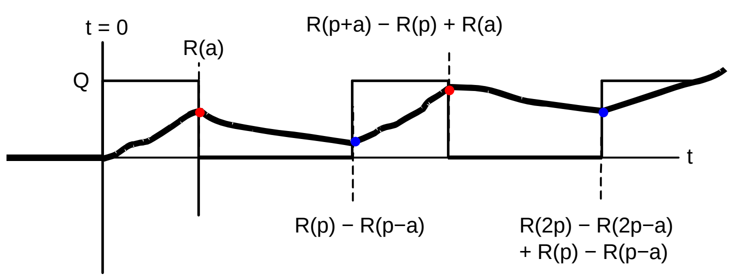


Figure 14. Response Starting from Ambient Initial Condition

From the beginnings of formulas tallied up in Figure 14, with careful attention to the summation ranges, we can generalize as follows: The steady−state peak temperature after the n^{th} pulse will be given by:

$$H(a,p,n) = \sum_{j=1}^{n} R[(j-1)p + a] - \sum_{j=1}^{n-1} R[jp] \quad \text{(eq. 10)}$$

and similarly, the steady−state “valley” temperature after the n^{th} complete cycle will be given by:

$$V(a,p,n) = \sum_{j=1}^{n} R[jp] - \sum_{j=1}^{n} R[jp-a] \quad \text{(eq. 11)}$$

The limiting cases will be found as n approaches infinity. For the peaks,

$$R(t,d) = \lim_{n \to \infty} H(t,p,n) \quad \text{(eq. 12)}$$

and for the valleys,

$$Y(t,d) = \lim_{n \to \infty} V(t,p,n) \quad \text{(eq. 13)}$$

One useful further refinement is to observe that the terms in the summations can be paired. In the “peak” temperature formula, since the first positive summation has one more term than the negative summation, we can choose whether to pull out separately the first term or the last one, and pair what’s left over. If we pull off the last (n^{th}) term of the first summation, we have:

$$H(a,p,n) = R[(n-1)p + a] + \sum_{j=1}^{n-1} R[(j-1)p + a] - \sum_{j=1}^{n-1} R[jp] = R[(n-1)p + a] - \sum_{j=1}^{n-1} \{R[jp] - R[(j-1)p + a]\} \quad \text{(eq. 14)}$$

So in the limit, as the number of terms approaches infinity (or we reach the end of the finite tabulation of R(t) and have therefore reached steady−state), we can say:

$$R(t,d) = R_{\infty} - \sum_{j=1}^{\infty} \{R[jp] - R[(j-1)p + a]\} \quad \text{(eq. 15)}$$

Let us now make the physically reasonable assumption that $R(t)$ is a monotonically increasing function (that is to say, it always gets hotter the longer you heat it). Then it must be true that each term of the summation is a positive number, i.e., because:

$$jp > (j-1)p + a$$

then

$$R[jp] > R[(j-1)p + a] \quad \text{(eq. 16)}$$

hence each term of the summation lowers the result with respect to value accumulated without it. Graphically, this formula can be interpreted as shown in the Figure 15. The peak temperature for any specific duty cycle can be obtained by taking the steady−state value, and subtracting the sum of all the portions of the single−pulse heating curve of the “off” periods for that specific pulse width.

Returning to Equation 10, our second option was to separate the $j = 1$ term from the first summation and pair the remainder like this (note the need to re−index the first summation):

$$\begin{aligned} H(a,p,n) &= R[a] + \sum_{j=2}^{n} R[(j-1)p + a] - \sum_{j=1}^{n-1} R[jp] \\ &= R[a] + \sum_{j=1}^{n-1} R[jp + a] - \sum_{j=1}^{n-1} R[jp] \quad \text{(eq. 17)} \\ &= R[a] + \sum_{j=1}^{n-1} \{R[jp + a] - R[jp]\} \end{aligned}$$

Again, if $R(t)$ is monotonically increasing, then:

$$R[jp + a] > R[jp] \quad \text{(eq. 18)}$$

and these restructured summation terms likewise are positive definite. Once again, in the infinite limit,

$$R(t,d) = R[a] + \sum_{j=1}^{\infty} \{R[jp + a] - R[jp]\} \quad \text{(eq. 19)}$$

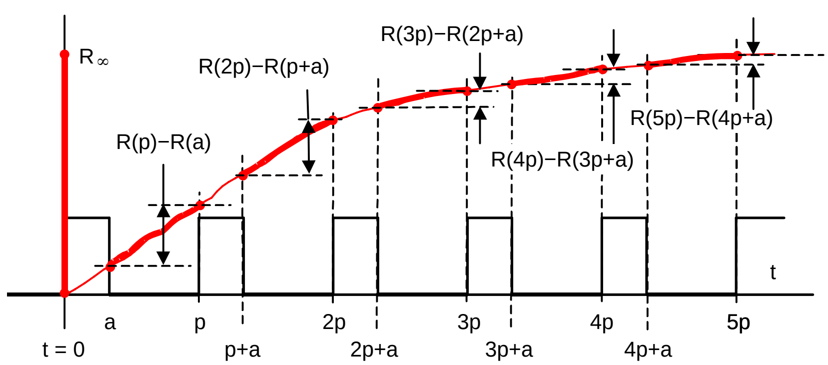


Figure 15. First Graphical Interpretation of “Peak” Duty Cycle Response

This formulation of the result has a particularly "intuitive" interpretation. Recall that we started with a system at ambient, that is, unpowered thermal equilibrium, and began to excite it with a square wave power input starting at time zero. Obviously R[a] is simply the single pulse response at the end of the first applied pulse. Then, each R[a + jp] is the single−pulse response at the *end* of the "on" pulse segment of each subsequent cycle, whereas each R[jp] is the single−pulse response at the *beginning* of each cycle (just before the associated pulse is turned on). This is essentially the complement of Equation 15, and the graphical interpretation is also the complement – i.e., instead of taking the total steady−state response and subtracting the spaces between the pulses, simply add up the spaces occupied by the pulses themselves, as shown in the Figure 16.

From this interpretation, it should be evident that as the period of the wave train gets small, each individual segment of the heating curve can be approximated by a small, straight line of width p along the time axis, hence each "on" element has length d•p. Thus (Figure 17), whatever vertical distance is spanned by the segment for each cycle, the vertical contribution of the "on" portion must be d times that span. Since the total vertical span for the entire heating curve is R_∞, then the total contribution of all segments to the peak duty cycle response must be $d \bullet R_\infty$. This short−period (or high frequency) "limiting value" of the duty cycle response is certainly no surprise. Yet it helps us intuitively grasp that even for very fast power cycling of a device, the quasi−steady−state peak temperature is determined not solely by the short−time region of the single−pulse heating curve. It of necessity includes substantial contributions of the curve (roughly a fraction of d) all the way out to the longest measured response times.

Similarly, as the period lengthens with respect to the shortest−time response measured (and even more specifically, as the pulse width a grows), the initial pulse consumes a larger and larger fraction of the total response curve, and the subsequent pulses obviously contribute less and less. We then can also see why the detailed shape of the curve will interact significantly with the period and the duty cycle, in forming the peak duty cycle response at times intermediate to the shortest and longest times.

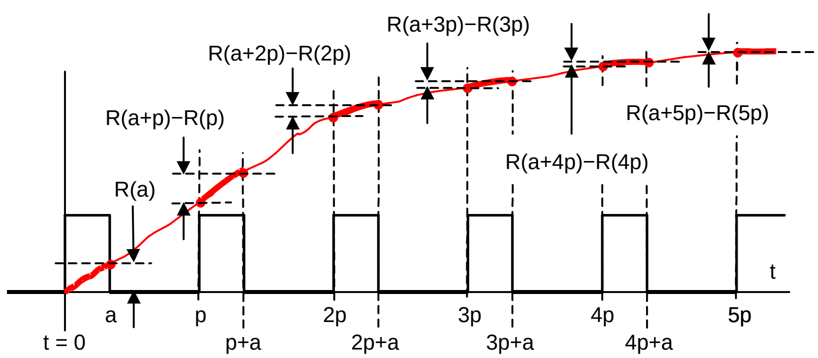


Figure 16. Second Graphical Interpretation of "Peak" Duty Cycle Response

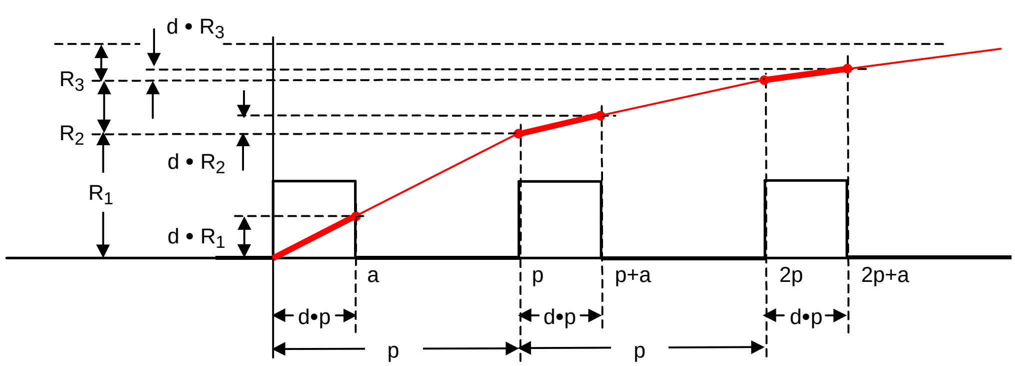


Figure 17. For Small p, Treating Heating Curve as Straight Line Segments

Though likely of less practical interest, we may follow similar reasoning to restate the "valley" response formula and graphically represent its construction from the single−pulse response in Figure 18. Note the significant difference between the solutions as far as the time−points of interest. For the "valley" response, we need to extract information from the single−pulse response at the beginning of each heating period (as before), but then at a pulse−width time *preceding* each pulse (refer again back to Figure 13), rather than *following* each pulse. From Equations 11 and 13, we quickly obtain:

$$Y(t, d) = \sum_{j=1}^{\infty} \{R[jp]-R[jp-a]\} \quad \text{(eq. 20)}$$

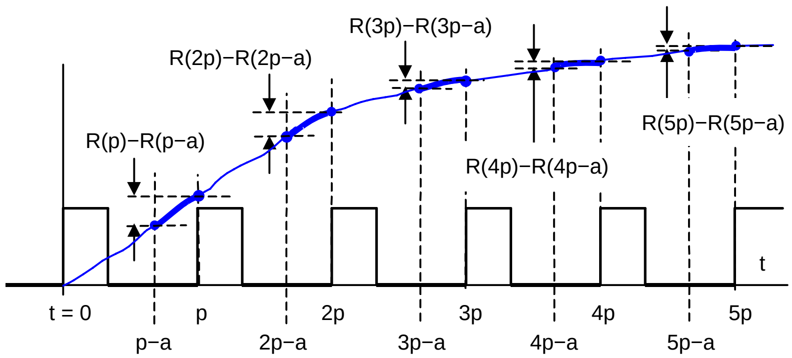


Figure 18. Graphical Interpretation of "Valley" Duty Cycle Response

Method 2 – Pulse train applied to system at constant power equilibrium

The second approach to generating the formula for peak temperature response is to begin at powered equilibrium. Unless the duty cycle is 100%, we can be assured that the peak temperature (i.e., the intercycle maximum) will be higher than the constant−power equilibrium temperature, and the valley temperature (i.e., the intercycle minimum) will be lower. According to the linear superposition principle, the temperature changes *relative to this equilibrium*, will be identical to those of the same cyclical power input having an average power of zero, as illustrated below.

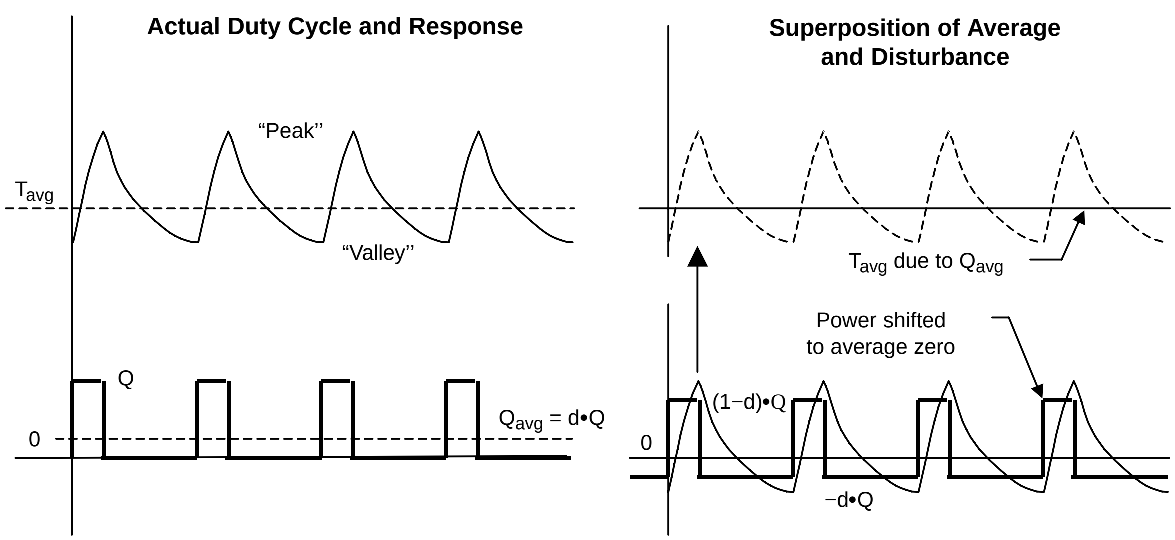


Figure 19. Equivalence of Non−Zero Average Response, and Superposition of Constant Average Plus Zero−Average Disturbance

We know that the temperature rise due to the average power will be given by:

$$d \cdot Q \cdot R_{\infty} \quad \text{(eq. 21)}$$

As before (see especially Example 2 above), we obtain the transient response to the zero−averaged−power cycling by a superposition of positive and negative−going power steps, as indicated in Figure 20.

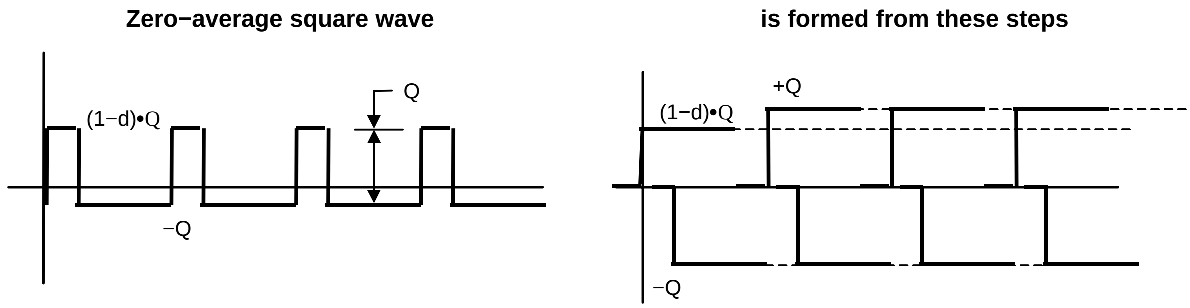


Figure 20. Construction of Zero−Power Averaged Square Wave from Steps

Consequently, just as we found in Example 2, we obtain the response by summing the step responses of the individual positive and negative going components. All but the initial step have equal amplitudes of $\pm Q$. Only the initial rising edge is different: that step has an amplitude of $(1-d)Q$. If all we desired was a first−order estimate of the peak temperature, therefore, all we would need to do is add the pulse response at time a to the average, that is:

$$T_1 = d \cdot Q \cdot R_\infty + (1-d) \cdot Q \cdot R(t) \qquad \text{(eq. 22)}$$

or, normalizing relative to the peak power (dividing through by Q)

$$R_1(t,d) = d \cdot R_\infty + (1-d) \cdot R(t) \qquad \text{(eq. 23)}$$

which is simply Equation 5, previously derived from “standard” Equation 1.

We can improve on this first order estimate by adding in the contributions of additional steps. For reasons which should become clear shortly, let us add these in by pairs, to keep things balanced. Let us step out to time $(p+a)$, and tally up the various contributions. From the initial step, we have:

$$(1-d) \cdot Q \cdot R(p + a) \qquad \text{(eq. 24)}$$

From the first negative−going step (which started at time a and has lasted one full period p), we have:

$$-Q \cdot R(p) \qquad \text{(eq. 25)}$$

and from the second positive going pulse which started at time p and has lasted for duration a:

$$-Q \cdot R(a) \qquad \text{(eq. 26)}$$

If we tally these up, plus the initial average temperature rise, we obtain:

$$T_2 = d \cdot Q \cdot R_\infty + (1-d) \cdot Q \cdot R(p + a) - Q \cdot R(p) + Q \cdot R(a) \qquad \text{(eq. 27)}$$

again dividing through by Q to yield units of thermal resistance, we get:

$$R_2(t,d) = d \cdot R_\infty + (1-d) \cdot R(p + a) - R(p) + R(a) \qquad \text{(eq. 28)}$$

which, except for the use of the parameters a and p (in lieu of t and d), and a reordering of the last two terms, is precisely Equation 6, which itself came from the “standard” Equation 2.

We have thus generated the formulas stated at the outset. Better, though, we see whence they arise, and can fairly easily generalize them. In particular, it should now be evident that we could continue to add in pairs of negative and positive going pulses, and calculate the peak temperature at the edge of the last pulse added in. For instance, we can quickly generate the expression taking us out to the third pulse:

$$R_3(t,d) = d \cdot R_\infty + (1-d) \cdot R(2p + a) - R(2p) + R(p + a) - R(p) + R(a) \qquad \text{(eq. 29)}$$

and extending the equation into a fully general form, we can now say:

$$R_n(t,d) = d \cdot R_\infty + (1-d) \cdot R[(n-1)p + a] + \sum_{j=1}^{n-1} \{R[(j-1)p + a] - R[jp]\} \qquad \text{(eq. 30)}$$

or, by reordering the terms under the summation and changing the external sign, we have:

$$R_n(t,d) = d \cdot R_\infty + (1-d) \cdot R[(n-1)p + a] - \sum_{j=1}^{n-1} \{R(jp) - R[(j-1)p + a]\} \qquad \text{(eq. 31)}$$

As before, with monotonically increasing $R(t)$ each summation term is positive definite, and thus we can state with certainty that all $R_n(t,d)$ for $n>1$ are smaller than the first−order estimate of the peak temperature from Equation 5 or 23. Indeed, every additional term will reduce further the estimate of the peak temperature. This conclusion is one important result which was sought from the outset.

Our final task is to explore the limit of Equation 31 as n goes to infinity. It should be clear that:

$$\lim_{n \to \infty} R[(n-1)p + a] = R_\infty \qquad \text{(eq. 32)}$$

therefore, utilizing this in the second term of Equation 31 as we replace the upper limit of the summation with infinity, we have:

$$R(t,d) = d \cdot R_\infty + (1-d) \cdot R_\infty - \sum_{j=1}^{\infty} \{R(jp) - R[(j-1)p + a]\} \quad \text{(eq. 33)}$$

$$R(t,d) = d \cdot R_\infty + R_\infty - d \cdot R_\infty - \sum_{j=1}^{\infty} \{R(jp) - R[(j-1)p + a]\} \quad \text{(eq. 34)}$$

so:

$$R(t,d) = R_\infty - \sum_{j=1}^{\infty} \{R(jp) - R[(j-1)p + a]\} \quad \text{(eq. 35)}$$

which is identical to Equation 19 derived earlier in Method 1. Thus, whether starting from uniform ambient or constant−power equilibrium, we find the same result in the infinite time limit.

Peak−to−Peak (Peak to Valley) Temperature Difference

Taking the difference between Equation 19, the peak temperature, and Equation 20, the valley temperature, we have a direct expression for the total temperature excursion experienced by the device when operating at steady−state under the influence of a constant duty cycle square wave.

$$R(t,d) = R[a] + \sum_{j=1}^{\infty} \{R[jp + a] - R[jp]\} \quad \text{(eq. 19)}$$

$$Y(t,d) = \sum_{j=1}^{\infty} \{R[jp] - R[jp-a]\} \quad \text{(eq. 20)}$$

so:

$$\Delta(t,d) = R[a] + \sum_{j=1}^{\infty} \{R[jp + a] - R[jp]\} - \sum_{j=1}^{\infty} \{R[jp] - R[jp-a]\} = R[a] + \sum_{j=1}^{\infty} \{R[jp + a] + R[jp-a] - 2R[jp]\} \quad \text{(eq. 36)}$$

It may be seen that if $R(t)$ changes relatively slowly and smoothly, then the terms of this summation should go to zero fairly quickly. Since the value of R is taken just before and just after each jp, the two values should average out to the value *at* each jp, and the summation terms become merely the "error" in this statement. For what it's worth, for small duty cycles (a small compared to p) the resulting summation may be recognized as essentially a central difference approximation for the second derivative of the single−pulse heating curve, that is to say:

$$\Delta(t,d) \approx R[a] + a^2 \sum_{j=1}^{\infty} \left. \frac{\partial^2 R(t)}{\partial t^2} \right|_{t = jp} \quad \text{(eq. 37)}$$

For slowly varying $R(t)$, this tells us what we probably already knew, namely that the peak−to−valley temperature excursions are approximated by the single−pulse response based on the pulse width.

Limitations of the Infinite Series Solution

What can be said about how many terms of Equation 31 or 35 are required in order to obtain a given accuracy? Unfortunately, very little. All we can say from either formulation is that we're starting with some initial approximate value and subtracting away the same series of values. In the case of (31), there are only a finite number of "corrections" (dictated by our choice of n, which also dictates how much lower is the starting value from R_∞). Since these are the same n corrections as the first n terms of the series in (35), and the starting value is lower for (31) than for (35), we conclude that it will take more terms of (35) to get to the same value we had in (31). We have little idea, however, of *how many* more terms of (35) it might take to get to the value of (31)'s finite approximation, nor how much lower (35)'s final value might go after an infinite number of terms are subtracted away. Without detailed numerical knowledge of the shape of the single−pulse curve, there is little else we can say. It is tempting to think that successive terms provide progressively smaller corrections. To a certain extent this must be true, because with each correction, there is yet less of the remaining difference left. However, we can never be sure that the very next term after the one we just computed won't consume the entire remainder of the difference (which might be considerably more than the magnitude of the term just before).

There is also the problem of numerical "noise." Unless we're working with a mathematical model of the thermal characteristics to begin with, any computation of the value $R[(j-1)p+a]$ requires that we make some assumption about the behavior of the function between discrete, tabulated values. Typically, a tabulated single−pulse response curve will have values at, say, 0.1, 0.2, 0.4, 0.7, and 1.0 milliseconds. What value do we use for 0.53 milliseconds? Do we linearly interpolate between 0.4 and 0.7, or logarithmically interpolate, or just what? And these tabulated values themselves will necessarily introduce round−off error at best, or experimental variation at worst, so our finite or infinite series summation of values interpolated between these discrete values will reflect peculiar accumulations of error or round−off at intervals harmonically related to the pulse width and period of interest. Numerical experimentation with real and hypothetical tabulated single−pulse curves demonstrates that however "smooth" and monotonic is the starting curve, the R_1 approximation tends to be similarly smooth. However, the R_2 and all higher order approximations tend to exhibit unrealistic wobbles in the computed response to other duty cycles − wobbles so large that the computed response may not even remain monotonically increasing! This includes effectively "infinite" series where the computations are carried out until every pair of terms ends up in the truly “steady−state” tail of the table. This also seems to hold true whether linear or logarithmic interpolation is performed between tabulated points. Can we get away with using only the R_1 approximation? Perhaps, though again, numerical experimentation with real transient response curves demonstrates that there can be as much as a 6% overestimate of $R(t,d)$ in R_1 over R_2.

The RC Network Solution

From the foregoing discussion, it seems that the amount of error and the possibility of accumulating numerical round−off, etc., in the series solution to this problem are not insignificant. It was suggested that some of this difficulty might be lessened if one were working with a mathematical model of the single−pulse curve rather than a tabulated, purely experimentally derived curve. In this next section, we will explore a specific mathematical model, namely an exponential series equivalent to the single−pulse response, earlier presented as Equation 4:

$$R(t) = \sum_{i=1}^{m} R_i\left(1-e^{-\frac{t}{\tau_i}}\right) \qquad \text{(eq. 4)}$$

Note that in the limit,

$$\lim_{t \to \infty} R(t) = \sum_{i=1}^{m} R_i \qquad \text{(eq. 38)}$$

There are two key features or advantages of this model over a tabulated function. First, we can make some precise statements about the way in which successive terms of the various formulas derived earlier fall off with time (i.e., they will fall off exponentially, which means the error can be bounded precisely, if it turns out necessary). Second, we need make no decision about interpolation methods, because this “decision” has already been made for us simply in the choice of the exponential function. We can compute the value of $R(t)$ to any precision we desire, at as precise a point in time as we desire. Because we will be able to derive a closed−form solution, however, we will see that neither the details of the error fall−off, nor the precision of the calculations, will be issues of any consequence at all.

What of the accuracy of the model with respect to experimental measurements, that is, the “fit” of the model to the data? Certainly if the fit is only good to within a few percent, we should not expect our computed square wave responses to be any better. However, if the fit of the model to the experimental data rivals the raw experimental noise level (generally quite feasible), then we should be able to rely on this exponential model to provide noise−free values of the response – and even better monotonicity of derived responses than we obtained with the series solutions based on tabulated data.

Closed Form of Peak Temperature for RC Network

Let us now pursue a closed−form solution to the fixed duty cycle square wave response based on the model of Equation 4. Because the series solutions derived previously consist simply of linear combinations of $R(t)$, it should be evident that we can momentarily simplify matters by considering but a single term (Equation 4). Whatever conclusions we reach from that analysis, we may then simply sum them up to obtain the total result. We start, therefore, with Equation 19, which constructed the solution as the sum of the portions of the single−pulse response occupied by the “on” pulse widths. Recall:

$$R(t,d) = R[a] + \sum_{j=1}^{\infty}\{R[jp+a]-R[jp]\} \qquad \text{(eq. 19)}$$

Let us also make explicit use of a previously suggested, but heretofore inconsequential, feature of the single−pulse response, namely that it begin at zero. Specifically, let us assume that for all real devices,

$$R(0) = 0 \qquad \text{(eq. 39)}$$

This is certainly a thermodynamically sensible requirement – until energy has been applied for a finite period of time, the temperature rise must remain zero. Making this stipulation, we can subtract Equation 39 from Equation 19,

$$R(t,d)-0 = R[a]-R(0) + \sum_{j=1}^{\infty}\{R[jp+a]-R[jp]\} \qquad \text{(eq. 40)}$$

and absorb the leading terms into the summation by extending the index to zero, yielding:

$$R(t,d) = \sum_{j=0}^{\infty}\{R[jp+a]-R[jp]\} \qquad \text{(eq. 41)}$$

We are now ready to substitute in for the function $R(t)$ a single term from Equation 4, thus:

$$R_i(t,d) = \sum_{j=0}^{\infty}\left\{R_i\left(1-e^{-\frac{jp+a}{\tau_i}}\right)-R_i\left(1-e^{-\frac{jp}{\tau_i}}\right)\right\} \qquad \text{(eq. 42)}$$

Expanding, collecting terms constant with respect to index j, and removing them from the summation, we obtain the following:

$$\begin{aligned} R_i(t,d) &= \sum_{j=0}^{\infty}\left\{R_i-R_ie^{-\frac{jp+a}{\tau_i}}-R_i+R_ie^{-\frac{jp}{\tau_i}}\right\} \\ &= \sum_{j=0}^{\infty}\left\{R_i\left(e^{-\frac{jp}{\tau_i}}-e^{-\frac{jp+a}{\tau_i}}\right)\right\} \\ &= \sum_{j=0}^{\infty}\left\{R_i\left(1-e^{-\frac{a}{\tau_i}}\right)e^{-\frac{jp}{\tau_i}}\right\} \\ &= \tau_i\left(1-e^{-\frac{a}{\tau_i}}\right)\sum_{j=0}^{\infty}e^{-\frac{jp}{\tau_i}} \end{aligned} \qquad \text{(eq. 43)}$$

Then utilizing the identity $\sum_{j=0}^{\infty} z^j = \frac{1}{1-z}$, our final expression for the contribution of each term of Equation 4 is:

$$R_i(t,d) = R_i \frac{1-e^{-\frac{a}{\tau_i}}}{1-e^{-\frac{p}{\tau_i}}} \qquad \text{(eq. 44)}$$

Summing them together for the entire m−term RC network response, we thus obtain:

$$R(t,d) = \sum_{i=1}^{m} R_i \frac{1-e^{-\frac{a}{\tau_i}}}{1-e^{-\frac{p}{\tau_i}}} \quad \text{or in terms of } (t,f) \text{ as:}$$

$$R(t,d) = \sum_{i=1}^{m} R_i \frac{1-e^{-\frac{t}{\tau_i}}}{1-e^{-\frac{1}{f\tau_i}}} \qquad \text{(eq. 45)}$$

This can also be written in terms of t and d, as in:

$$R(t,d) = \sum_{i=1}^{m} R_i \frac{1-e^{-\frac{t}{\tau_i}}}{1-e^{-\frac{t}{d\tau_i}}} \qquad \text{(eq. 46)}$$

Observe that as t gets small, because $1-e^{-x} \rightarrow x$, this formula reduces to:

$$\lim_{t \rightarrow 0} R(t,d) = \sum_{i=1}^{m} R_i \frac{\frac{t}{\tau_i}}{\frac{t}{d\tau_i}} = \sum_{i=1}^{m} R_i d = d \sum_{i=1}^{m} R_i \qquad \text{(eq. 47)}$$

So if the sum of the amplitudes of the RC network terms equals the steady−state thermal resistance of the system (as it should, if the model is any good), then we have the expected limit for high frequency response of the solution. Further, at the other extreme (as t gets large), all the exponential terms vanish, and the solution degenerates as it should to exactly:

$$\lim_{t \rightarrow \infty} R(t,d) = \sum_{i=1}^{m} R_i \qquad \text{(eq. 48)}$$

Closed Form of "Valley" Temperature for RC Network

In an almost identical manner we can generate the closed form solution for the intercycle minimum temperature of the fixed duty cycle square wave. Starting with Equation 20:

$$Y(t,d) = \sum_{j=1}^{\infty} \{R[jp]-R[jp-a]\} \qquad \text{(eq. 20)}$$

and substituting in a single term of the RC solution for $R(t)$, we have:

$$Y_i(t,d) = \sum_{j=1}^{\infty} \left\{ R_i\left(1-e^{-\frac{jp}{\tau_i}}\right) - R_i\left(1-e^{-\frac{jp-a}{\tau_i}}\right) \right\} \qquad \text{(eq. 49)}$$

With minor differences, we manipulate as before to this point:

$$Y_i(t,d) = R_i\left(e^{\frac{a}{\tau_i}}-1\right) \sum_{j=1}^{\infty} e^{-\frac{jp}{\tau_i}} \qquad \text{(eq. 50)}$$

and now utilize another identity, $\sum_{j=1}^{\infty} z^{-j} = \frac{1}{z-1}$, to obtain the expression:

$$Y_i(t,d) = R_i \frac{e^{\frac{a}{\tau_i}}-1}{e^{\frac{p}{\tau_i}}-1} \qquad \text{(eq. 51)}$$

or, explicitly in terms of t and d,

$$Y_i(t,d) = R_i \frac{e^{\frac{t}{\tau_i}}-1}{e^{\frac{t}{d\tau_i}}-1} \qquad \text{(eq. 52)}$$

Then, finally, summing over all the terms of Equation 5, we have:

$$Y_i(t,d) = \sum_{i=1}^{m} R_i \frac{e^{\frac{t}{\tau_i}}-1}{e^{\frac{t}{d\tau_i}}-1} \qquad \text{(eq. 53)}$$

Unfortunately, with the positive exponential terms, there may be computational limitations. It therefore is much more numerically robust to rewrite the equation as follows*:

$$Y(t,d) = \sum_{i=1}^{m} R_i e^{\left(1-\frac{1}{d}\right)\frac{t}{\tau_i}} \frac{1-e^{-\frac{t}{\tau_i}}}{1-e^{-\frac{t}{d\tau_i}}} \qquad \text{(eq. 54)}$$

or in terms of period and on−time:

$$Y(t,d) = \sum_{i=1}^{m} R_i e^{\frac{a-p}{\tau_i}} \frac{1-e^{-\frac{a}{\tau_i}}}{1-e^{-\frac{p}{\tau_i}}}$$

*For instance, the largest argument permitted for the exponential function in the 2003 release of Excel is 709, resulting in a value of approximately 1E308. In an RC network model whose smallest time constant is 1E−6, we will therefore reach the 71st term and thus exceed the computational capability of Excel before we've gotten to even 1E−3! However, Excel will happily accept −1E307 as a valid argument for the same function, returning the value 0.

where each exponential argument is negative definite. As with the peak temperature expression, for high frequency waves we expect the valleys to approach the average temperature. To confirm this, here we observe that as t gets small, $e^{x}-1 \rightarrow x$, so this formula reduces to:

$$\lim_{t \rightarrow 0} Y(t,d) = \sum_{i=1}^{m} R_i \frac{\frac{t}{\tau_i}}{\frac{t}{d\tau_i}} = \sum_{i=1}^{m} R_i d = d \sum_{i=1}^{m} R_i \quad \text{(eq. 55)}$$

On the other hand, as t gets large, the exponential terms dominate, leading to:

$$\lim_{t \rightarrow \infty} Y(t,d) = \sum_{i=1}^{m} R_i \frac{e^{\frac{t}{\tau_i}}}{e^{\frac{t}{d\tau_i}}} = \sum_{i=1}^{m} R_i \left(e^{\frac{t}{\tau_i}}\right)^{-\left(\frac{1}{d}-1\right)} \quad \text{(eq. 56)}$$

Therefore, for $d < 1$ the outer exponent is negative, the inner term gets large, hence:

$$\lim_{t \rightarrow \infty} Y(t,d) = 0 \quad \text{(eq. 57)}$$

which we should expect. In other words, for any finite duty cycle less than 100%, if you make the cycle's period long enough (by making the pulse width large enough), the device will cool back to ambient during the off portion of each cycle, i.e., the valley temperature is ambient.

Finally, if $d = 1$, we see immediately from Equation 53 that:

$$\lim_{t \rightarrow \infty} Y(t,0) = \sum_{i=1}^{m} R_i \quad \text{(eq. 58)}$$

This, also, we expect. It means that for a 100% duty cycle (power is never turned off), we reach the steady−state thermal resistance and the device never cools at all.

As our last exercise in utilizing the RC model for the square wave analysis, let us compute the peak−to−valley steady−state excursions. To do so, subtract Equation 54 from Equation 46 to obtain:

$$\Delta(t,d) = \sum_{i=1}^{m} R_i \left[1 - e^{\left(1-\frac{1}{d}\right)\frac{t}{\tau_i}}\right] \frac{1-e^{-\frac{t}{\tau_i}}}{1-e^{-\frac{t}{d\tau_i}}} \quad \text{(eq. 59)}$$

RC Predictions for Generalized Square Wave

It was shown in the previous section that an RC model can be summed over an infinite number of cycles to obtain the peak and valley temperatures of the "steady−state" system. A straightforward application of the same method allows us to use the RC model to predict the temperature at any point in time of a steady−state system excited by a periodic square wave. Hence we may also calculate the temperature at any time within a cycle after an infinite number of cycles of *any* periodic waveform (as in Figure 10). First, consider the "generalized" square wave shown in Figure 21.

Two things distinguish this from the square wave input (Figure 12) previously analyzed for the duty cycle chart. First, the pulse is not assumed to turn on at the beginning of the cycle (i.e., we need to be able to place a component of a more general waveform anywhere within a cycle); second, we would like to be able to compute the response of the system at any time t during the cycle, not necessarily at any particular edge of any particular component of the input waveform. If we first consider the fraction of the general cycle where t is greater than b (hence also greater than a), and (as before) recognize that the response after an infinite number of cycles is the cumulative summation of all the "on" pulses minus the cumulative summation of all the "off" pulses (all relative to the same position within a cycle), we can write:

$$F(a,b,p) = \sum_{j=0}^{\infty} R[t-a+jp] - \sum_{j=0}^{\infty} R[t-b+jp] \quad \text{(eq. 60)}$$

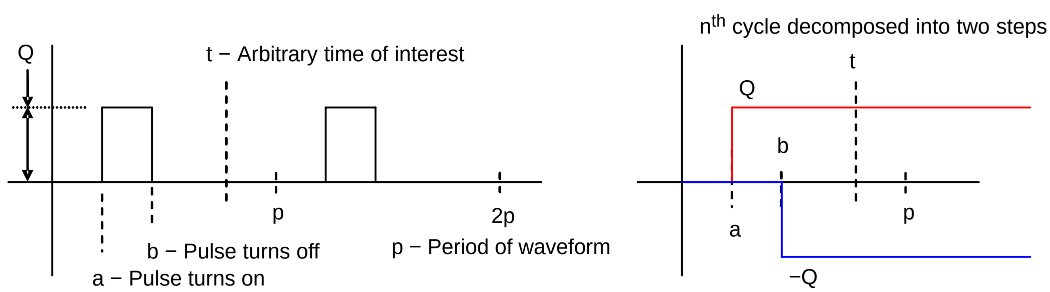


Figure 21. Generalized Square Wave

For clarity, we now replace the transient response $R(t)$ with just a single term of the RC network expansion (which will subsequently be summed over all terms of the network), to obtain:

$$F_i(a, b, p, t, i) = R_i \sum_{j=0}^{\infty} \left[\left(1 - e^{-\frac{t-a+jp}{\tau_i}} \right) - \left(1 - e^{-\frac{t-b+jp}{\tau_i}} \right) \right]$$

$$F_i(a, b, p, t, i) = R_i \sum_{j=0}^{\infty} \left[e^{-\frac{jp}{\tau_i}} \left(e^{\frac{b-t}{\tau_i}} - e^{\frac{a-t}{\tau_i}} \right) \right] \quad \text{(eq. 61)}$$

$$= R_i \left(e^{\frac{b-t}{\tau_i}} - e^{\frac{a-t}{\tau_i}} \right) \sum_{j=0}^{\infty} \left(e^{-\frac{p}{\tau_i}} \right)^j$$

$$= R_i \left(\frac{e^{\frac{b-t}{\tau_i}} - e^{\frac{a-t}{\tau_i}}}{1 - e^{-\frac{p}{\tau_i}}} \right) \quad \text{(eq. 62)}$$

So now summing over all the terms of the RC network model, we have:

$$F(a, b, p, t) = \sum_{i=1}^{m} R_i \left(\frac{e^{\frac{b-t}{\tau_i}} - e^{\frac{a-t}{\tau_i}}}{1 - e^{-\frac{p}{\tau_i}}} \right) \text{ good for } b \le t \le p,$$

computable for all $t \ge b$. (eq. 63)

Indeed, if Equation 63 is used, as is, to compute results for values of $t>p$, what one finds is that the cumulative response keeps falling until eventually it returns to zero, as t increases from p to infinity. The problem is that once the infinite sum has been executed, the original periodicity

$$F(a, b, p + t, t) = F(a, b, p, t) \quad \text{(eq. 64)}$$

is merely implied, as there is nothing, in effect, to turn the pulse back on again after it has turned off at time $t=b$. (To emphasize, observe that Equation 63 is now but a *finite* sum over a finite RC model; the *infinity* of cycles and their periodicity has been consumed in the denominator, which no longer has any explicit periodicity.)

Now, as was discussed previously, potentially positive exponential arguments (as in the numerator of Equation 63) cause computational difficulties. Clearly, so long as $t>b$ (hence $t>a$), Equation 63 yields negative exponentials and everything is computationally smooth, and it correctly predicts the net transient response for all times greater than b, on up through time $p+a$ (which is when the loss of true periodicity in Equation 63 first affects the results). But the solution of Equation 63 for the range $p \le t < p+a$ is precisely the response we seek for the period between $0 \le t < a$, which is where we have positive exponential arguments in Equation 63. So the simple modification of Equation 63 by subtracting p from the numerators' exponential arguments, shifts the solution from the beginning of the next cycle back to the beginning of the nominal cycle. We may thus state:

$$F(a, b, p, t) = \sum_{i=1}^{m} R_i \left(\frac{e^{\frac{b-t-p}{\tau_i}} - e^{\frac{a-t-p}{\tau_i}}}{1 - e^{-\frac{p}{\tau_i}}} \right) \text{ good (computable)}$$

only for $0 \le t < a$. (eq. 65)

The remaining range that must be considered is $a \le t < b$. It will be left as an exercise to the reader to show that the following correctly handles the situation:

$$F(a, b, p, t) = \sum_{i=1}^{m} R_i \left(1 + \frac{e^{\frac{b-t-p}{\tau_i}} - e^{\frac{a-t}{\tau_i}}}{1 - e^{-\frac{p}{\tau_i}}} \right) \quad \text{(eq. 66)}$$

good (computable) only for $a \le t < b$.

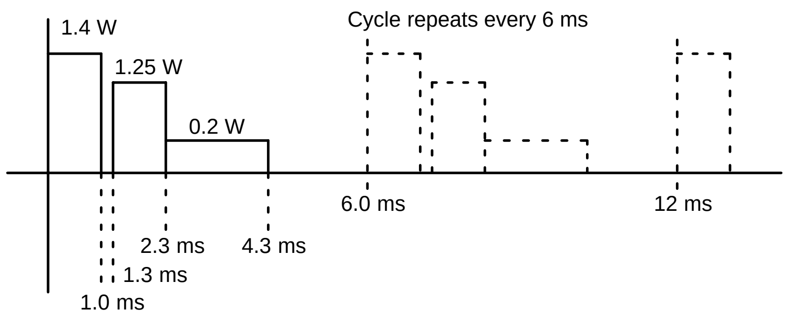
RC Predictions for Steady−State of Arbitrary Waveform

We may now expand this generalized single square pulse solution into a more complex application of power. If a general periodic waveform is constructed from square pulses (all having different a's and b's, and magnitudes, Q, but a common p), the principle of linear superposition gives us:

$$G(t) = \sum_{k=1}^{n} Q_k F(a_k, b_k, p, t) \quad \text{(eq. 67)}$$

We may thus make a direct computation of the steady−state temperature profile resulting from *any* periodic excitation (refer back to Figure 10), assuming that one is able to satisfactorily approximate the desired periodic waveform with a series of square pulses.

Consider the following example, with the periodic power input of Figure 22 applied to the RC model given in the table to the right.



tau (sec)	R [°C/W]
1.00E−6	0.01104
1.00E−5	0.012806
1.00E−4	0.069941
1.00E−3	0.275489
1.00E−2	0.019806
1.00E−1	1.128566
1.00E+0	3.539626
1.00E+1	5.423616
1.00E+2	12.08694
1.00E+3	16.2933

Figure 22.

Applying Equations 65, 66, and 63 to each of the three separate portions of each of three separate square pulses comprising the repeated pattern, and Equation 66 to superimpose their effects, we find the following temperature response:

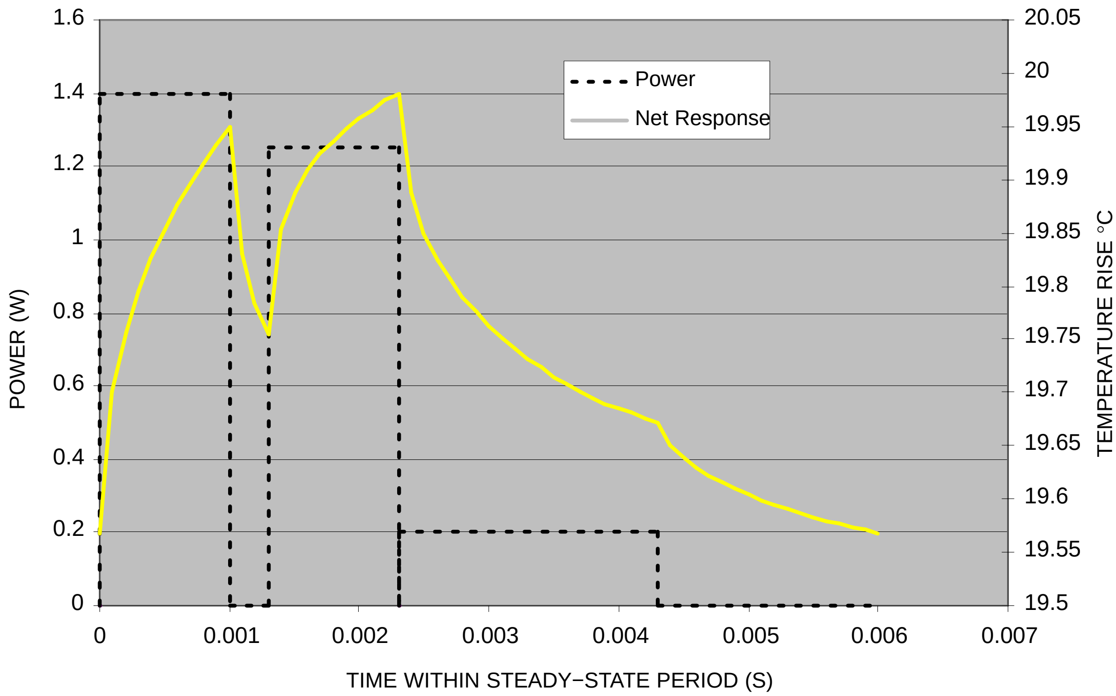


Figure 23.

What makes this example particularly interesting is that the peak temperature (during a steady−state cycle) occurs at the end of the *second* pulse, which has a lower power, and even a small gap of zero power, between it and the higher power pulse immediately preceding it in the cycle. Knowing that the single pulse response is proportional to power, and that the peak temperature always occurs at the end of a square pulse (even when infinitely repeated), might easily lead one to overlook the possibility demonstrated here. In other words, for a generalized periodic waveform, even when constructed of (or perhaps approximated by) just a small number of square sub−components, one does well to compute the response through the entire range of a cycle, not at just the “obvious” points.

Multiple Interacting Junctions

Just as the linear superposition technique allows us to calculate the temperature response of a single−junction device to relatively complicated power inputs, starting with the basic single−pulse heating curve, we can use linear superposition to predict temperature response of a junction to multiple other heat sources. The main additional complexity which must be introduced, is that we must have single−pulse heating responses for each interaction of interest. That is, not only the response of each junction to its own heating, but the response of each junction to step−heating applied to every other powered junction in the system. Then the temperature at any given junction at any specific time, is the linear superposition of its response to the power applied at all junctions (including itself) summed up over all preceding time history. If different junctions are "turned on" at different times, we simply offset the contributions in time, just as we did when a single junction's response was determined by adding or subtracting its own single−pulse responses accumulated over the entire preceding time history. Likewise, as junctions are turned off, their heating responses, scaled and subtracted from the accumulating total, contribute just as they did in the single−junction case. The bookkeeping is obviously a little more challenging, but there is nothing different in principle. It should be easy to see, however, that as the power inputs get "interesting," this method will become cumbersome. For instance, powering two junctions with square wave trains, if the two are of differing frequency, cannot be handled simply by superimposing the duty cycle curves developed in isolation (even when the interaction heating curve is known). These sorts of problems are much better suited to a SPICE type simulation utilizing the thermal RC model of the system.

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